A Constraint Optimization Approach to Causal Discovery from Subsampled Time Series Data

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California Institute of Technology
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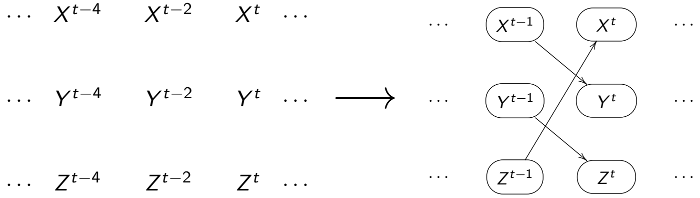
20.9.2016

Problem Statement

How to discover the causal structure at the **system timescale** from time series data obtained at a coarser **measurement timescale**?



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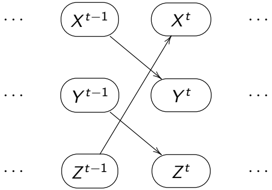

- Only every u:th vector of values is observed (**subsampling rate** u)

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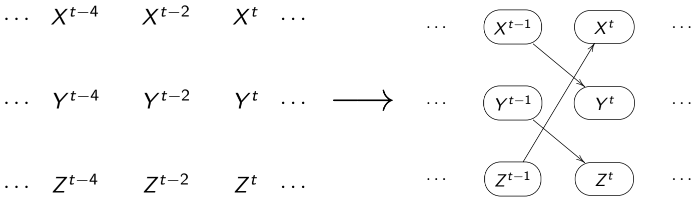

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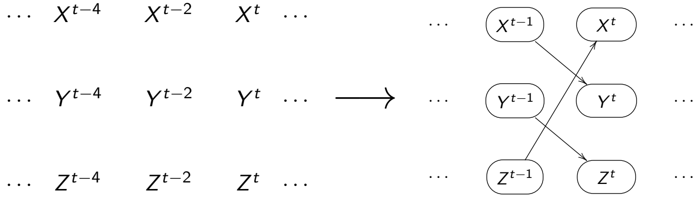
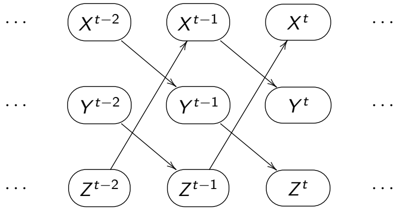
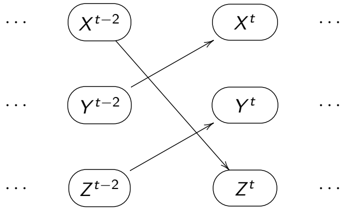


- Only every u:th vector of values is observed (**subsampling rate** u)
- Subsamping induces confounding, and unidentifiability
- Applications: e.g. fMRI.

Subsampling needs to be taken into account!



True structure at the system timescale



measurement time scale structure

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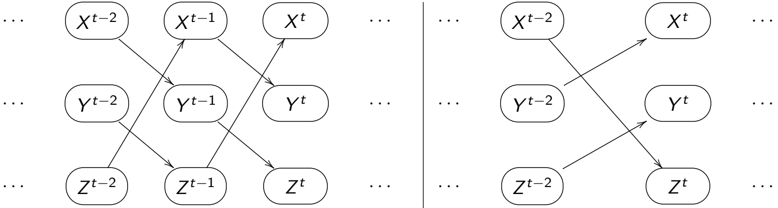

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When ignoring subsampling:

- All direct causal relationships misspecified.

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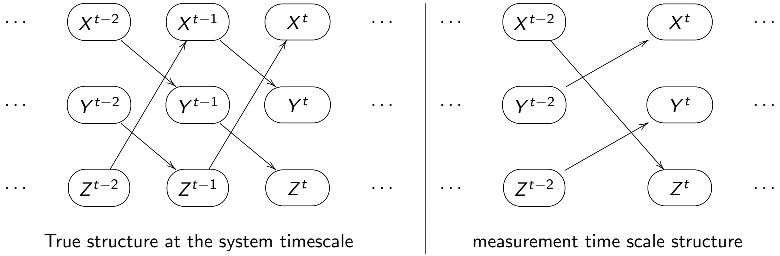

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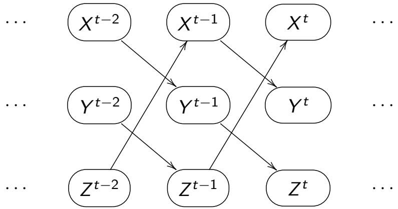

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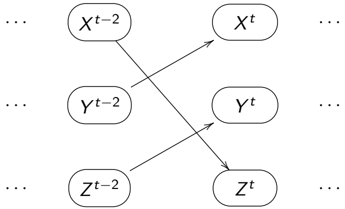

measurement time scale structure

When ignoring subsampling:

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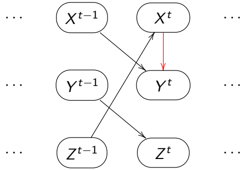
- Wrong interventions suggested.

Overview

Previous Literature

- Adding instantaneous effects in a linear model (see for example Lütkepohl 2005 or Hyvärinen et al 2010).



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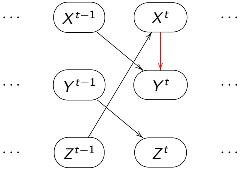

- Continuous time approaches, but some processes are inherently discrete time (e.g. salary payment).

Previous Literature 2

Recently Plis et al. (UAI2015,NIPS2015) considered modeling subsampling directly, assuming on the system timescale level:

- discrete time
- first order Markov: $\mathbf{V}^t \perp\!\!\!\perp \mathbf{V}^{t-k} | \mathbf{V}^{t-1}$
- no instantaneous effects, or unobserved common causes
- nonparametric (continuous or discrete values, SVAR processes, or dynamic BNs)
- Measurements from this at integer intervals (e.g. every second).

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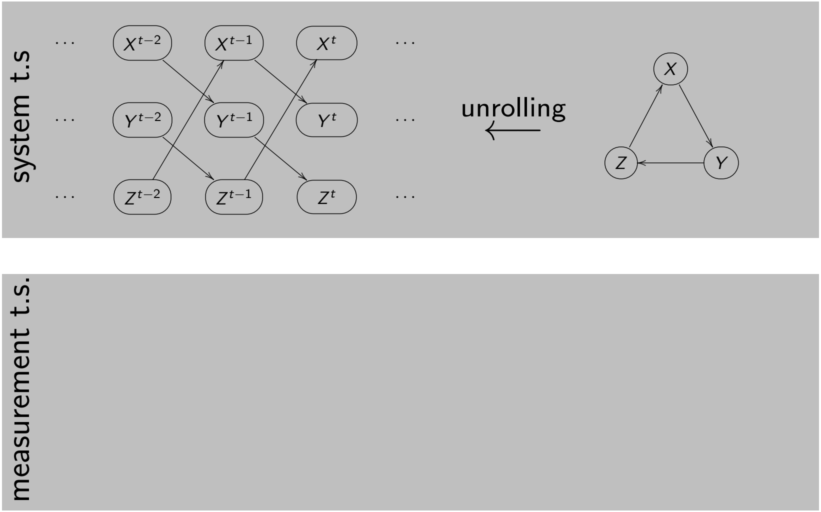
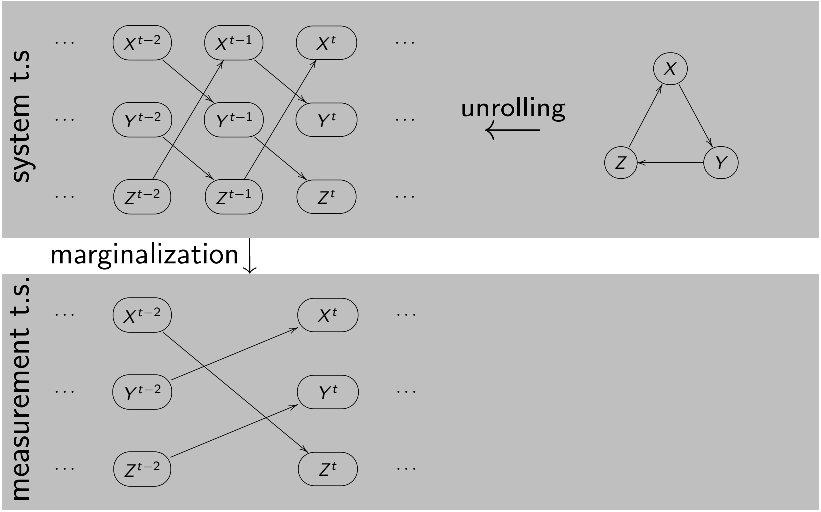
Corresponding parametric method: Gong et al. (ICML2015) discovered linear models using non-Gaussianity.

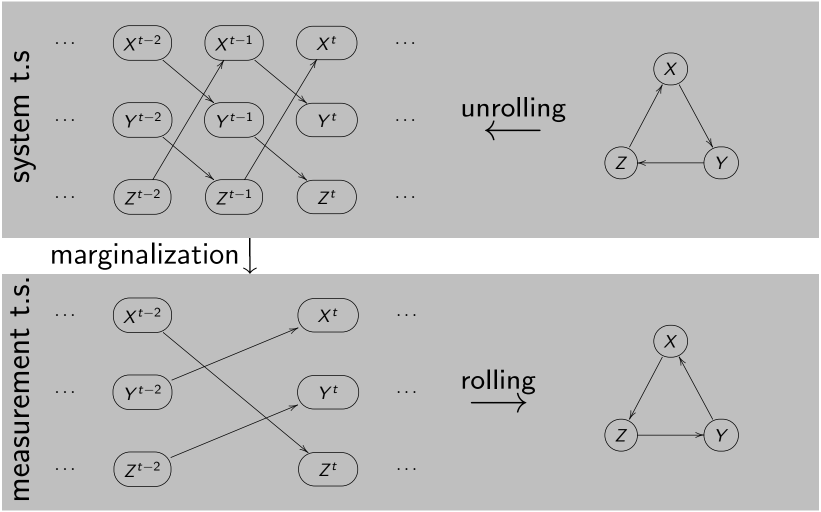
Graphical Representation

Rolled Representation

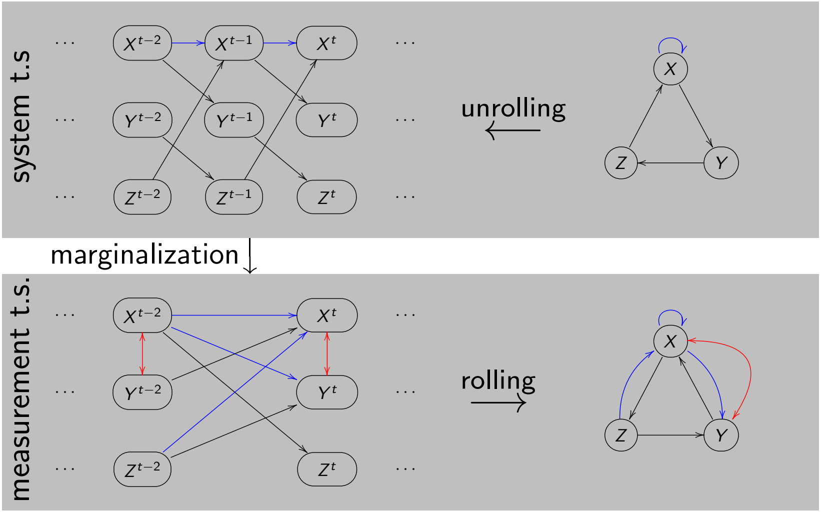



system t.s
...
X^{t-2}
X^{t-1}
X^t
Y^{t-2}
Y^{t-1}
Y^t
Z^{t-2}
Z^{t-1}
Z^t
unrolling
X
Z
Y
marginalization
measurement t.s.
X^{t-2}
X^t
Y^{t-2}
Y^t
Z^{t-2}
Z^t

Rolled Representation



Induced confounding



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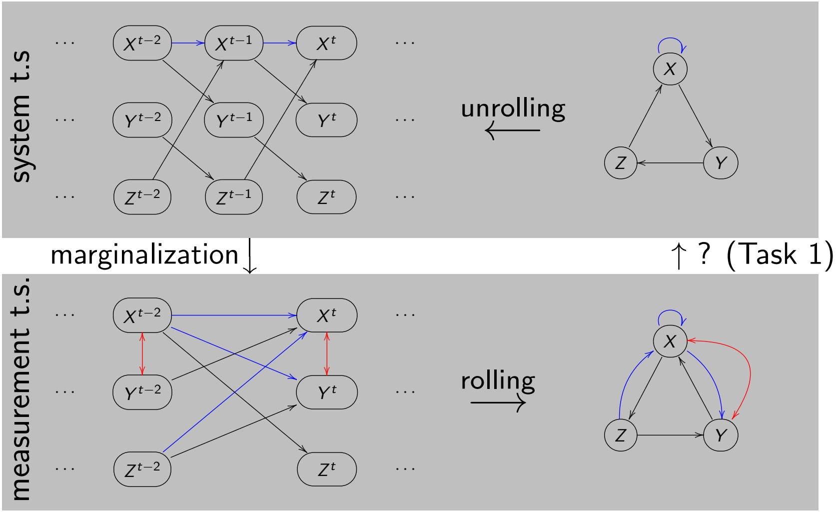
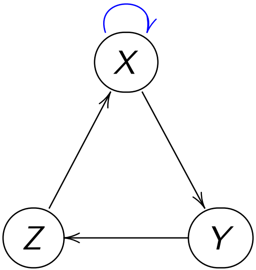
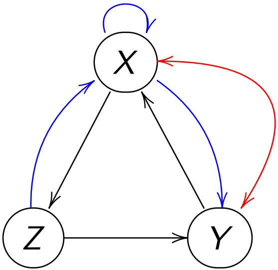


Correspondence between System and Measurement T.S.



system timescale



measurement timescale

When subsampling by u:

- Measurement time scale edge $Y \to X$ corresponds to path of length u: $Y \to \cdots \to X$
- Measurement time scale edge $X \leftrightarrow Y$ corresponds to paths of length $k < u$: $W \to \cdots \to X$ and $W \to \cdots \to Y$.

A Constraint Satisfaction Solution

Partial Complexity Result

Result: Deciding whether there is a system t.s. structure compatible with the directed edges of a measurement t.s. structure is **NP-complete** for any fixed $u \geq 2$.

Proof: Binary matrix root.

A Constraint Satisfaction Solution by ASP

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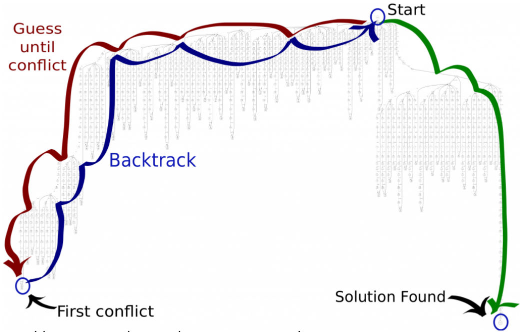
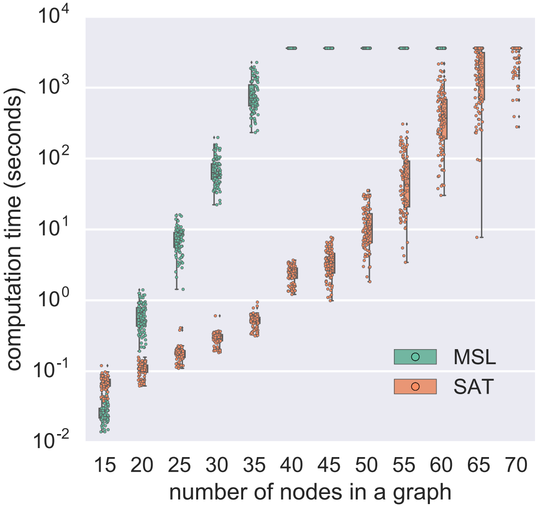
- You write a symbolic encoding.
- The symbolic encoding gets grounded.
- The encoding gets turned into conjunctive normal form.
- Backtracking DFS by `Clingo` (Gebser et al. 2011).
- Exact and complete solution.
- Subsampling rate *u*: fixed or free.



https://srlabs.de/bites/minisat-intro/

```
node(1..3). % Measurement timescale structure
edgeh(1,2).no_edgeh(1,3).confh(2,3).no_confh(1,2). %and so on

urange(1..5).  % Define a range of u:s
1 { u(U): urange(U) } 1. % u(U) is true for only one U

{ edge1(X,Y) } :- node(X), node(Y). %draw G1

% Derive all directed paths up to length U
path(X,Y,1) :- edge1(X,Y).
path(X,Y,L) :- path(X,Z,L-1), edge1(Z,Y), L <= U, u(U).

% Determine measurement t.s. for G1
edgeu(X,Y) :- path(X,Y,L), u(L).
confu(X,Y) :- path(Z,X,L), path(Z,Y,L), node(X;Y;Z),
                               X < Y, L < U, u(U).
% Check consistency
:- edgeh(X,Y), not edgeu(X,Y). :- no_edgeh(X,Y), edgeu(X,Y).
:- confh(X,Y), not confu(X,Y).  :- no_confh(X,Y), confu(X,Y).
```

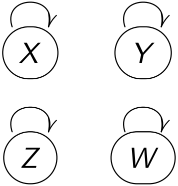
Scalability of Enumerating 1000 Solutions



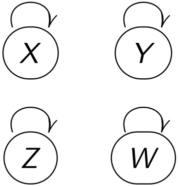
(fixed subsampling rate 2, SAT is our approach,
MSL is the previous state of art by Plis et al. (2015))

Identifiability: Underdetermination

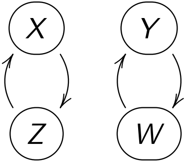
Measurement timescale structure:



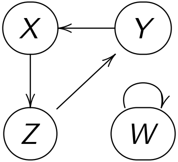
could be produced by system timescale structures:



$u = 1, 2, 3, \cdots$



$u = 2, 4, 6, \cdots$

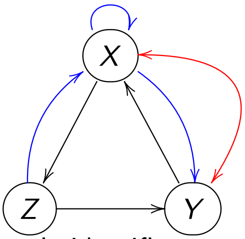


$u = 3, 6, 9, \cdots$

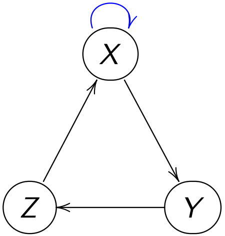
or a four cycle in either direction and symmetrically!

Identifiability: An Identified Case

But measurement timescale structure:



uniquely identifies system timescale structure



and the subsampling rate $u = 2$.

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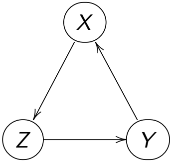

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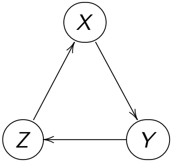

data | measurement t.s. | system t.s.

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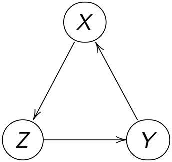

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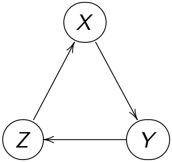

data — measurement t.s. — system t.s.

- Measurement t.s. structure can be consistently estimated from data under faithfulness: e.g.

$$X \rightarrow Z \quad \Leftrightarrow \quad X^{t-u} \not\perp\!\!\!\perp Z^{t} \mid \mathbf{V}^{t-u} \setminus X^{t-u}$$

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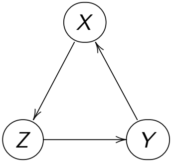

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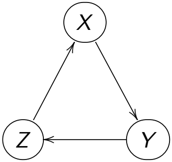

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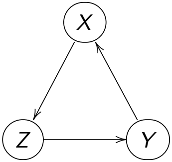

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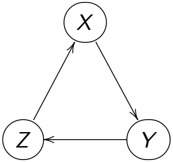

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- Due to finite samplesize, the constraint satisfaction approach will often return UNSATISFIABLE.
- Find the system t.s. structure such that its measurement t.s. structure is optimally close to the estimated (Task 2).

A Constraint Optimization Solution by ASP

Specifics:

- Penalize inconsistencies between absences and precences of edges in the measurement t.s.:
  - Either uniform weights, or
  - log Bayesian probabilities of the corresponding (in)dependence, obtained through Bayesian model selection (see Hyttinen et al. 2014)
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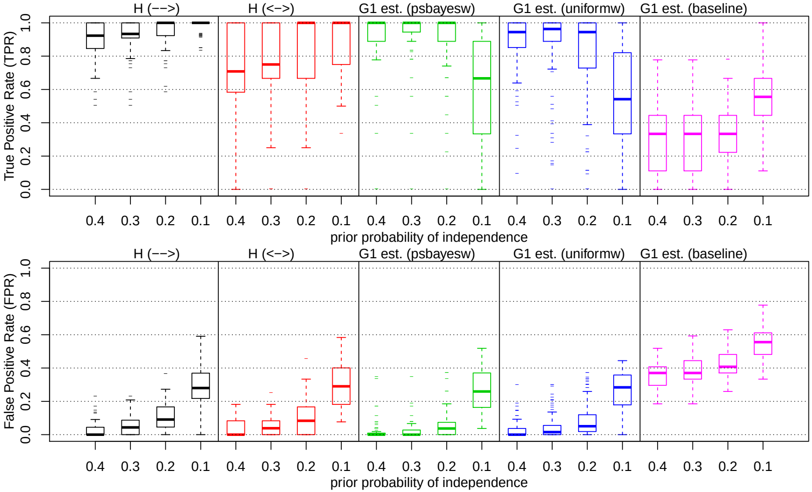
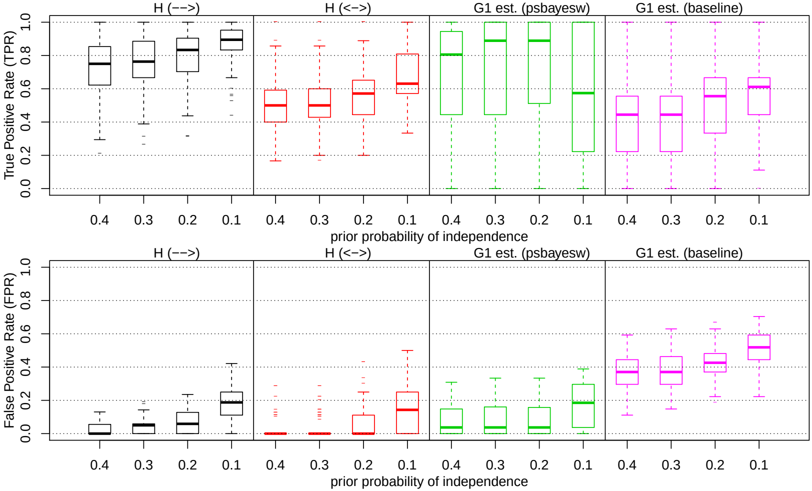
Accuracy for fixed $u = 2$



(fixed subsampling rate 2, average result of the eq. class,
6 nodes, av. degree 3, 200 samples, 100 data sets, linear models)

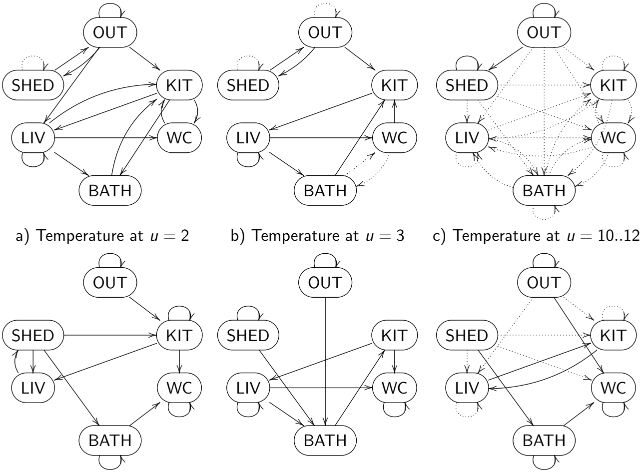
Accuracy for $u = 3$



Analysis of Temperature/Humidity data 1

- Hourly measurements of six sensors placed in a house.
- Temperature and humidity recorded.
- Removed trends.
- Handle undetermination: for each edge [Magliacane et al.]
  - run the inference procedure enforcing presence
  - and then enforcing absence
  - difference in objectives gives the support for the edge.

Analysis of Temperature/Humidity data 2



a) Temperature at $u = 2$ b) Temperature at $u = 3$ c) Temperature at $u = 10..12$

d) Humidity at $u = 2$ e) Humidity at $u = 3$ f) Humidity at $u = 10..12$

Edges with full lines are found to be present, absent edges are found to be absent, edges with dotted lines are present or absent.

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Thanks!